

## **A Status Report on Chiral Dynamics: Theory and Experiment**

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### **Abstract**

This paper gives an overview of a Workshop on Chiral Dynamics: Theory and Experiment which was held at MIT July 25-29, 1994; its unique feature was the equal mixture of theory and experiment. The purpose of the workshop was to bring together many of the active participants to assess the status of the field and to explore fruitful future directions. The foundations and present status of the theory were reviewed as well as the experimental status of the field. To facilitate the discussion of future directions working groups were organized on Threshold Photo/Electropion Production, Two-Pion Threshold and the Chiral Anomaly, Nucleon Polarizabilities, Pion and Sigma Polarizabilities,  $\pi - \pi$  Interactions, and  $\pi$ -N Interactions.

The main attraction of chiral dynamics [Weinberg, Leutwyler, We79,Ga83]\* is that it represents a rigorous and model-independent methodology by which to make QCD predictions at the confinement scale. This is a forefront arena of the standard model and as such it is essential to confront these predictions with precise experiments. Interest in this field has been growing rapidly due to increasing confidence in the theory and also as a result of a new generation of accelerators and experimental techniques. Chiral dynamics, or chiral perturbation theory, is an (low energy) effective field theory for QCD. At the confinement scale where the interaction between the quarks is very strong the relevant degrees of freedom are the baryons  $N, \Sigma, \Delta$ , *etc.* and the Goldstone bosons  $\pi, \eta, K$ . A perturbative treatment is appropriate since the Goldstone interactions are relatively weak at low energies. Indeed the Goldstone theorem requires the vanishing of all interactions of Goldstone bosons at zero energy-momentum. The effective field theory obeys all of the underlying symmetries of the QCD Lagrangian. Of particular relevance is chiral symmetry, which is broken both dynamically and also explicitly by the small but non-zero light quark masses  $(m_u, m_d, m_s)$ .

Isospin is another important approximate symmetry which was discussed by both Weinberg and Leutwyler. At a superficial level one might expect large isospin violations due to the rather large ratio  $r = (m_d - m_u)/(m_d + m_u) \sim 0.3$ . However, the observed isospin violation is much smaller due to the fact that the up and down quark masses are small compared to the QCD scale and the relevant isospin breaking quantity is

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\* References to talks at the workshop shall be denoted by the name of the author in parenthesis. The summary of the workshop will be published by Springer-Verlag as *Chiral Dynamics: Theory and Experiment*, A.M. Bernstein and B.R. Holstein, editors. Whenever possible these will be supplemented by references to the published literature.

$(m_d - m_u)/\Lambda_{\text{QCD}} \sim 0.01$  [Leutwyler], which is of the same order as electromagnetic effects. Nevertheless there are some special cases which involve the s-wave  $\pi^0$  nucleon scattering or charge exchange reactions (in which the  $\pi^0$  appears in either the final and/or initial states) for which this isospin violation can be larger [We79], and this possibility, which is the focus of a new experimental initiative, is discussed below.

The price that must be paid for dealing with an effective, nonrenormalizable theory is the appearance of a number of unknown low energy constants, which must be determined experimentally. In principle they can also be obtained from the QCD Lagrangian by integrating out the high energy degrees of freedom, and it has been shown that their magnitudes can be reliably estimated by saturating with the low lying resonances [Ecker, Ec89] or very approximately determined by lattice gauge calculations [Negele, My94]. A related approach, which appears promising, is based on a chiral version of the Schwinger-Dyson equation [Roberts, Ro94]. In this model an ansatz must be made for the gluon distribution function. However, after that is done observables can be calculated without any additional parameters.

The predictions of chiral perturbation theory are expressed as a power series in energy-momentum and in current quark mass with a scale parameter  $\sim 4\pi F_\pi \sim 1 \text{ GeV}$ . The convergence properties of the theory are determined by the structure of the process under consideration and in a scattering problem, for example, will depend on the position of the lowest resonances and on the thresholds for particle production. The most accurately predicted quantities are the properties of the Goldstone bosons and their strong and electroweak interactions. In this sector there is a unique connection between the number

of loops and the corresponding power of the energy (momentum). Although most existing results are one loop or  $\mathcal{O}(p^4)$ , in a few cases two loop— $\mathcal{O}(p^6)$ —calculations are becoming available.

In the relativistic treatment of the  $\pi N$  sector the simple relationship between the number of loops and the power of energy (momentum) is not valid due to the presence of the nucleon mass as an additional parameter with the dimensions of energy. However, in the heavy fermion version of the theory the simple power counting is restored. There has been impressive progress in this arena in the past few years, as discussed by Meißner and Bernard[Be95a].

From the previous discussion one can observe a sort of “complementarity” about the experimental tests of chiral dynamics. On the one hand it is the Goldstone boson sector of the theory which is most amenable to reliable calculations. However, since one cannot make targets of these unstable particles more difficult, indirect methods, must be employed in order to confront these predictions with experimental tests. The most mature example, summarized by Počanić, is the low energy  $\pi - \pi$  interaction which was first predicted using current algebra and then refined with chiral perturbation theory. The original experimental technique used the nucleon as a source of virtual pions, with the  $\pi - \pi$  interaction determined via extrapolation to the pole at  $t = m_\pi^2$  (here  $t$  is the invariant four momentum transfer, which is negative in the physical region, hence the extrapolation). An alternative approach involves measurements of the final state  $\pi - \pi$  interaction in the near-threshold  $\pi N \rightarrow \pi\pi N$  reaction, for which there exists a significant data base. The largest error at the present time in the latter method is associated with the phenomenological

method used in order to extract the  $\pi - \pi$  scattering lengths from the data. The  $\pi - \pi$  interaction has also been measured in the  $K^+ \rightarrow \pi^+ \pi^- e^+ \nu_e$  decay ( $K_{e4}$  decay, branching ratio  $\sim 3.9 \times 10^{-5}$ ). At the present time there appears to exist a discrepancy between results obtained via different methods [Počanić], as shown in Figure 1. However, due to possible systematic errors in the extraction of the scattering lengths from the  $\pi N \rightarrow \pi \pi N$  reaction and the low statistics of the  $K_{e4}$  decay data, it is premature to draw definitive conclusions. At the present level of precision, it is clear that more needs to be done. We can look forward to improved theoretical analysis of the  $\pi N \rightarrow \pi \pi N$  reaction and to vastly improved  $K_{e4}$  decay data from the Frascati  $\Phi$  factory DAΦNE [Gasser and Sevier, Baldini and Pasqualucci, Da92]. In addition, there are plans to measure the  $\pi - \pi$  scattering length by observation of ponium (bound  $\pi^+ \pi^-$  atoms) in experiments at CERN and Indiana.

An additional area in which we can look forward to new experimental results is the study of the chiral anomaly. In this case parameter-free predictions can be made for threshold reactions. An anomaly is said to occur for a situation in which a symmetry of the classical Lagrangian is not obeyed when the theory is quantized. The most famous and successful example in particle physics is the prediction for the  $\pi^0 \rightarrow \gamma \gamma$  decay rate. We can anticipate new results for the  $\gamma \pi \rightarrow \pi \pi$  reaction, for which there also exists a solid prediction based on the chiral anomaly. Although this prediction strictly speaking obtains only at the unphysical center of mass energy  $\sqrt{s} = 0$ , it can with reasonable assumptions be extended into the physical domain. There exists an approved experiment at CEBAF to study this reaction using virtual pions from a proton target in the reaction  $ep \rightarrow \pi^+ \pi^0 ne'$  as well as at Fermilab, which will utilize a high energy pion beam and the virtual, *i.e.*

*Primakoff effect*, photons from a high Z nuclear target [Miskimen, Moinester].

Hadron polarizabilities  $\bar{\alpha}$  and  $\bar{\beta}$  (electric and magnetic), which measure the response of systems to the presence of external electromagnetic fields, are important probes of internal structure. Again the “complementarity” between theory and experiment applies in that the predictions for the (hard to measure) pion are considerably more precise than for the nucleon, for which abundant targets are available. The nucleon situation was summarized by Nathan. For the proton, which has been studied for over 30 years, there has been considerable recent progress[Ha93]. At the present time the most precise results are obtained using the forward scattering (unitarity) sum rule constraint for  $\bar{\alpha} + \bar{\beta}$  while fitting  $\bar{\alpha} - \bar{\beta}$  from the Compton scattering measurements. For the neutron the only precise data has been obtained from elastic neutron scattering from Pb combined with a sum rule constraint[Sc91]. The results are compared to theoretical predictions in Table 1. The  $\mathcal{O}(p^4)$  chiral perturbation theoretical results [Meißner] are shown with errors which take into account uncertainties associated with the relevant low energy constants. It can be seen that for  $\bar{\beta}$  (the magnetic polarizability), where the contribution of the  $\Delta$  is significant, the theoretical uncertainties are correspondingly large. Within errors, however, there is reasonable agreement between theory and experiment. Although this is encouraging, it is clear that more precise data, particularly on the neutron, is highly desirable. It is also very important to clarify the relationship between the formulations of baryon chiral perturbation theory with and without the  $\Delta$  as an explicit participant, and to reduce the theoretical uncertainty [Butler and Nathan].

**Table I.** Nucleon Polarizabilities ( $10^{-43} \text{ fm}^3$ )

	Experimental <sup>†</sup>	Theory*
$\bar{\alpha}_p$	$12.0 \pm 0.9$	$10.5 \pm 2.0$
$\bar{\alpha}_n$	$12.5 \pm 2.50$	$13.4 \pm 1.5$
$\bar{\beta}_p$	$2.2 \pm 0.9$	$3.5 \pm 3.6$
$\bar{\beta}_n$	$3.3 \pm 2.7$	$7.8 \pm 3.6$
$\bar{\alpha}_p + \bar{\beta}_p$	$14.2 \pm 0.5$	$14.0 \pm 2.2$
$\bar{\alpha}_n + \bar{\beta}_n$	$15.8 \pm 1.0$	$21.1 \pm 2.2$

\*CHPT [Meißner]      <sup>†</sup>[Nathan, Ha93,Sc91]

In the case of pion polarizabilities, much attention has been focussed upon the  $\gamma\gamma \rightarrow \pi^0\pi^0, \pi^+\pi^-$ , channels which will be studied at DAΦNE. In the  $2\pi^0$  case, discrepancies between SLAC experimental measurements and a one loop chiral perturbation theory calculation have recently been resolved via a dispersion relation estimate as well as a full two loop calculation, as shown in Figure 2[Gasser,Be94a]. A similar two-loop calculation is underway for the corresponding  $\pi^+\pi^-$  process. However, in this case the one-loop chiral and dispersive calculations are already in good agreement with each other and with experiment. At the present time there exist several experimental measurements of the pion polarizabilities, which are, however, highly divergent. The working group on hadron polarizabilities [Baldini and Bellucci] discussed three active areas for future measurements. These are: 1) the  $\pi^+Z \rightarrow \gamma\pi^+Z$  reaction from virtual photons in a high Z target (Primakoff effect) using high energy pions at Fermilab ( $K^+$  and  $\Sigma^+$  polarizabilities will be measured in a similar fashion) [Moinester]; 2) the  $\gamma p \rightarrow \gamma\pi^+n$  reaction (radiative photoproduction) extrapolated to the pion pole planned at Mainz; and 3) the  $e^+e^- \rightarrow e^+e^-\pi^+\pi^- (\pi^0\pi^0)$  reaction planned at DAΦNE [Baldini and Pasqualucci]. It has been demonstrated that

the latter reaction is an excellent probe of the dynamics but is unfortunately relatively insensitive to the pion polarizabilities [Gasser, Baldini and Bellucci, Do93]. We can look forward to results from each of these experimental initiatives in the next few years.

The  $\pi N$  interaction at low energies is a fundamental testing ground for any theory of the strong interactions. Of particular interest for chiral dynamics is the sigma term which is predicted to be  $\sigma = (35 \pm 5) \text{ MeV}/(1 - y)$  where  $y$  is a measure of the strange quark content of the nucleon [Sainio, Ga91]. There has been a long history of determining  $\sigma$  from the data which was reviewed by Sainio and Höhler. The complications include 1) an extrapolation must be made from the physical region to the Cheng-Dashen point by an analytic continuation of the empirically determined partial wave scattering amplitudes; 2) the relevant amplitude is isoscalar which is relatively small; 3) there exist systematic discrepancies in the data base; and 4) there is a rapidly varying  $t$ -dependence of the scalar form factor of the nucleon, which is determined by dispersion relations (lowest order chiral perturbation theory calculations are insufficient). To illustrate the problems associated with point 2 and 3 above, the sensitivity of the differential cross section to the sigma term for the elastic scattering of 30 MeV pions from the proton is shown in Figure 3 [Sa94]. It can be seen that the variation of the cross section, for a range in the sigma term corresponding approximately to the present uncertainty in its determination, is about the same as the systemic error between the data sets. Despite all of these difficulties the value  $y = 0.2 \pm 0.2$ , corresponding to a contribution of 130 MeV to the nucleon mass, has been determined [Sainio, Ga91]. It is clearly desirable to reduce the 100% uncertainty in  $y$ . Of related interest is the suggestion by Leutwyler to measure the  $\pi\pi$  sigma term from the



observation of the  $\pi - \pi$  scattering lengths in the ponium atom.

The interesting possibility of observing isospin violation due to the mass difference of the up and down quarks was discussed by Weinberg and Leutwyler. There was also a review of the present status of isospin violation in the  $\Delta$  region [Höhler] as well as the presentation of several recent developments. In particular, new measurements of the  $\pi^-p$  and  $\pi^-p \rightarrow \pi^0n$  charge exchange scattering lengths have been performed at PSI by observing the transition energy and width of the  $3p \rightarrow 1s$  transition in pionic hydrogen [A. Badertscher *et al.* in the  $\pi N$  working group summary]. The values obtained show a possible violation of isospin symmetry [Achenauer *et al.* in the  $\pi N$  working group summary]. There is also a proposed new technique by which to measure  $\pi^+n \rightarrow \pi^0p$  and possibly  $\pi^0p$  scattering lengths in the  $\gamma p \rightarrow \pi^0p$  reaction as a final state interaction effect [Be94]. All of the scattering lengths depend on the value of the  $\pi N$  coupling constant as was discussed by Pavan [ $\pi N$  working group summary] and Höhler.

Threshold electromagnetic pion production is an additional testing ground for chiral dynamics [Bernard, Meißner and Schoch, Be95b]. The field dates back to current algebra derivations of “low energy theorems” for threshold pion photoproduction. However, it has only been in the past five years that accurate experimental data have been available. A flurry of activity was spurred by experiments at Saclay and Mainz on the threshold  $p(\gamma, \pi^0)$  reaction [Ma86, Be90]. When the dust settled down it was found that these experiments agreed with the soft pion predictions [Be91]. However, it was subsequently realized that the “low energy theorems” omitted important loop contributions and at the workshop Ecker demoted them to “low energy guesses.” It now appears that the situation is more

complicated with a substantial fraction of the prediction for the threshold electric dipole amplitude  $E_{0+}$  coming from an undetermined low energy constant which must be determined from experiment [Bernard,Be95b]. Its value cannot be predicted with precision at the present time. Despite this situation, a *new* low energy theorem has been derived for two of the three p wave threshold multipoles [Bernard,Be95b], and these predictions agree with the Mainz data. However, since an unpolarized cross section does not uniquely determine the multipoles this does not constitute a proof of these predictions are correct. In fact there is another empirical solution for the Mainz data which disagrees with the new p wave numbers [Be94]. A definitive determination of the multipoles awaits the results of more recent experiments for the (unpolarized) cross sections performed at Mainz [St90] and SAL [Bergstrom] as well as an experiment planned with linearly polarized photons at Mainz.

It was pointed out by Bernard that once the relevant low energy constants are determined by the data for the  $p(\gamma, \pi^0)$  reaction, there are no additional free parameters in the predictions for the corresponding  $p(e, e'\pi^0)$  process. It is therefore of increasing importance to make precision measurements of the threshold electroproduction reaction. Two preliminary results were reported at the workshop [Blok, Walcher, Va95] from NIKHEF and Mainz for  $q^2 = -0.1 \text{ GeV}^2$  and for CM energies  $W$  up to 15 MeV above threshold. At Mainz the longitudinal and transverse contributions to the cross sections have been obtained (Rosenbluth separation). The results of this analysis will be available soon.

As the magnitude of  $q^2$  increases so does the relative contribution of the one loop contribution, reaching 50% of the total cross section by  $q^2 = -0.1 \text{ GeV}^2$ . Therefore it is

suggested that precision measurements of the  $p(e, e'\pi^0)$  reaction be carried out at a smaller magnitude of  $q^2$  [Bernard]. An experiment at  $q^2 = -.05 \text{ GeV}^2$  is planned at Mainz during the next year.

The previous discussion has been focused on neutral pion production. For charged pions one expects the old low energy theorems to be more accurate since the leading (Kroll-Ruderman) term for the threshold electric dipole amplitude  $E_{0+}$  comes from a marriage of gauge and chiral invariance and is model-independent. Nevertheless the existing experimental tests are based on older emulsion measurements and involve relatively large extrapolations to threshold. There are plans to perform the  $p(\gamma, \pi^+)n$  reaction at Saskatoon [Bergstrom]. Also preliminary results for the  $p(\pi^-, \gamma)n$  reaction near threshold were presented by Kovash for a TRIUMF experiment. Finally we note the emergence of the  $(\gamma, 2\pi)$  reaction as an interesting object of study of study [Bernard, Walcher]. Here preliminary measurements in the  $\pi^0\pi^0$  channel by the TAPS detector at Mainz have revealed a rather substantial signal in the near-threshold region.

The entire discussion of the  $\pi N$  sector is based on the physics of light (almost massless) up and down quarks. In the purely mesonic sector it is customary to treat the *three* light quarks (u, d, s). This SU(3) treatment has yet to be extended to the meson-baryon sector which would then include  $\pi, \eta, K$  as well as nucleons, hyperons (and  $\Delta, \Sigma^*$  ?). Measurements in this extended sector are already in progress at Bates where the SAMPLE experiment is looking for the strange quark content of the nucleon [Kowalski]. Likewise experiments at Bonn and Mainz and planned experiments at CEBAF are investigating kaon and eta photo- and electro-production. In the mesonic sector we also look forward

to exciting new data on the  $\pi - \pi$  interaction from  $K_{e4}$  decays and from the two photon production of pion pairs at DAΦNE [Baldini and Pasqualucci].

Although, in the interest of brevity, we have not discussed all of the new experimental and theoretical initiatives that are applicable to QCD studies at the confinement scale we hope that we have conveyed some of the sense of excitement and promise that was exhibited at the workshop. We anxiously look forward to future progress in this field.

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**Figure 1:** The  $I=2$  vs.  $I=0$  s-wave the  $\pi\pi$  scattering length predictions (symbols) and experimental results (contour limits). The “soft pion” results are from the near threshold  $\pi N - \pi\pi N$  reaction[Po94]; the “Chew-Low” results are from the peripheral  $\pi N - \pi\pi N$  reaction[Al82]; and the “ $K_{e4}$ +Ror” results are from the  $K_{e4}$  decay data using the constraint of the Roy equations[R077]. Dashed line: Weinberg’s constraint. Calculations: Weinberg (full square), Schwinger (filled triangle), Chang and Gursey (filled inverted triangle), Jacob and Scadron (open circle), Gasser and Leutwyler-ChPT (open square), Ivanov and Troitskaya-QLAD (open triangle), Lohse *et al.*-Meson Exchange (open rhomb), Ruivo *et al.* and Bernard *et al.*-NJL (open stars), Kuramashi *et al.*-quenched lattice gauge QCD (open cross), Roberts *et al.*-GCM (filled stars).

**Figure 2:** The  $\gamma\gamma \rightarrow \pi^0\pi^0$  cross section  $\sigma(|\cos\theta| \leq Z)$  as a function of the center of mass energy  $E$  at  $Z=0.8$ , together with the data from the Crystal Ball experiment[Ma90]. The solid line is the full two-loop result, and the dashed line results from the one-loop calculation[Gasser,Be94a]. The band denoted by the dash-dotted lines is the result of a dispersive calculation by Pennington[Pe92].

**Figure 3:** Differential cross section for  $\pi^+p$  elastic scattering at  $p_\pi = 97$  MeV/c (30 MeV pion kinetic energy). The curves show the variation of the predicted cross sections for a variation of  $\Sigma$  of 10 MeV which is the range of uncertainty in the  $\pi N$  sigma term[Sa94]. The points show the experimental data.